EVENT TIME

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Abstract

Investors take for granted that returns are recorded in units of time, such as days, months, or years. Yet some time periods include unusual events that reasonably cause asset prices to change, whereas other periods are relatively free of unusual events, in which case returns mostly reflect noise. Based on insights from information theory, the authors rescale time into event units so that each return is related to a common degree of event intensity. Their analysis reveals that when returns are measured in event units, their distributions are more normal and their co-occurrences are more stable, which enables analysts to form more reliable inferences.

EVENT TIME

We measure time as a function of how long it takes the earth to rotate on its axis and to travel around the sun. Then we use this system of measurement to record outcomes, which in some cases makes sense and in others does not. It seems reasonable, for example, to measure speed in units of time, such as how many seconds it takes a sprinter to run 100 meters. But does it make sense to measure how many points an NBA basketball player scores in a week? Not necessarily. If the week is in August, when no games are played, it makes no sense. And even if the week is in February, we would still want to standardize our measurement to account for the number of games the player participated in that week. It therefore makes sense to substitute events for time when recording certain outcomes.

This consideration invites the question of how we should record asset returns. What is the significance of the circumnavigation of the earth around the sun to an investment strategy? Why then do we focus so much on returns that are recorded in multiples or fractions of years? Implicitly, we assume that common units of time include the same frequency and intensity of relevant events. This assumption might make sense for multi-year periods owing to the law of large numbers, but it is less likely to hold for shorter periods such as days, months, or quarters. We propose that it might be more insightful to record asset returns in units of event intensity in which event intensity refers to a common degree of the frequency and significance of relevant events.

We proceed as follows. We first review a key insight of Claude Shannon, the father of information theory, to show how information relates to the probability of events. We then describe how we measure event intensity. Next, we show how the conversion from calendar time to event time leads to more normally distributed returns and more stable co-occurrences. We then discuss the implications of measuring returns in event time rather than calendar time on stress testing, performance evaluation, and portfolio construction.

Information and the Probability of Events

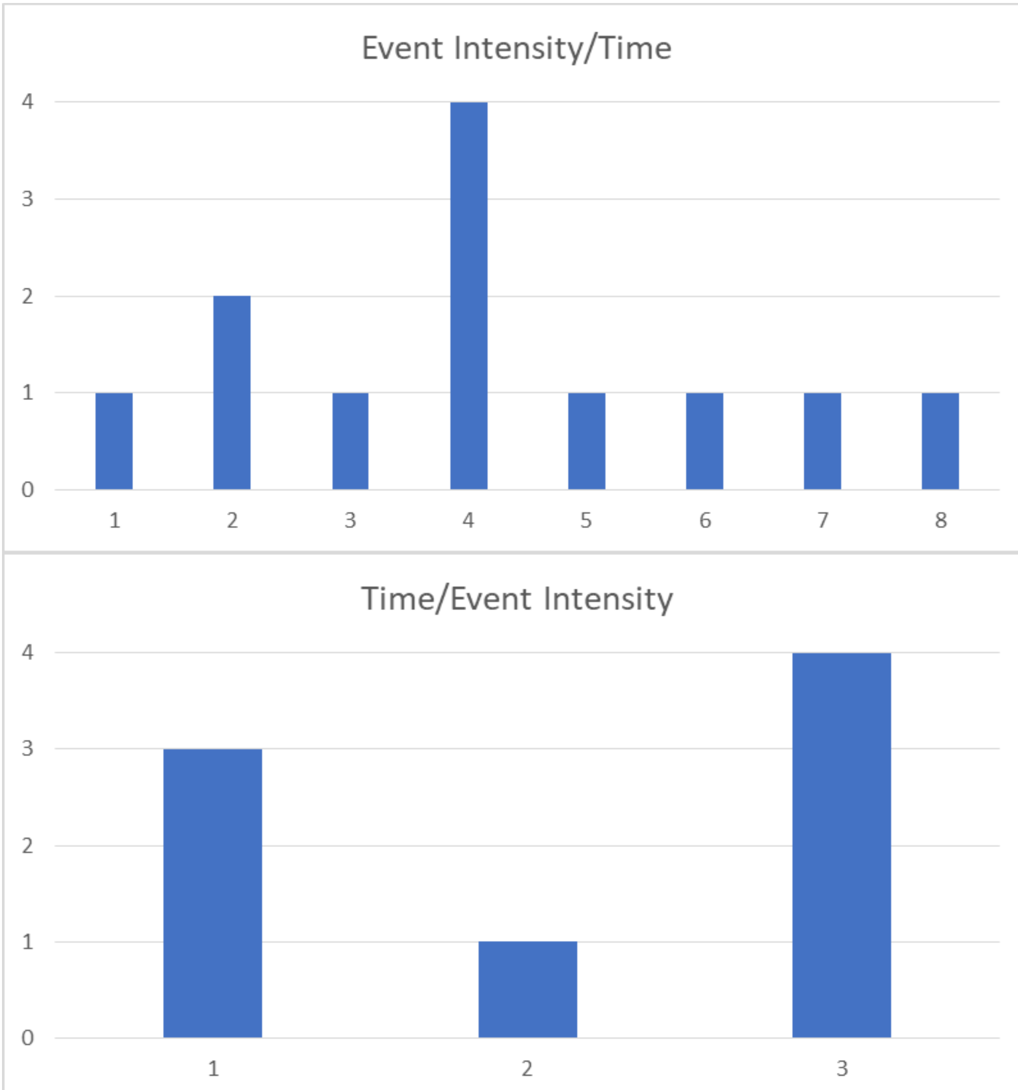
In 1948 while working in the mathematics department at Bell Labs, Claude Shannon published one of the most important scientific papers ever, "A Mathematical Theory of Communication," which is universally acknowledged as the founding of information theory. Shannon deduced from basic principles that information is inversely related to probability, which is to say that unusual events contain more information than common events.

To explain Shannon's insight, we begin by noting that the probability of an event equals the number of ways the event can occur divided by the total number of possibilities for all events. Shannon showed that when the probability of an event shrinks, we gain information and when the probability expands, we lose information. We therefore know more about events that have only a few ways of occurring than we do about events that have many ways of occurring. This inverse relationship between information and possibility sits at the core of Shannon's information theory.

Consider, for example, the sum of a random sequence of the numbers 1 through 10. It would be extremely unusual for the sum to equal 100, because of the 10 billion possible sequences there is only one way for the numbers to sum to 100. Therefore, we know everything there is to know about this sequence if its sum is 100. Now suppose the sum equals 99. This outcome is also unlikely, but not nearly as unlikely as 100. There are 10 ways the sum could equal 99. We would still know quite a bit about the sequence—that at least one number in the sequence is 9 and the others are 10. But we would not know which of the 10 numbers in the sequence 9 is. We therefore lose one piece of information. Now consider a sum of 98. To get 98, we need one number to be 8 or two numbers to be 9. There are 10 ways for a number to be 9, and for each of those we have nine ways for another number to be 9. Half of these 90 combinations are redundant, however, so we are left with 45 possibilities. Add to this outcome 10 ways for one number to be 8. In total, there are 55 ways to arrive at a sum of 98. Compared to the sum of 99, we now have one less piece of information because we know that two of the numbers (or one of the numbers, twice) must be incrementally lower than 10, but we do not know which numbers in the sequence are the lower numbers. The bottom line is that probabilities multiply and information adds. They are intimately connected. The lower the probability of an event, the more information it conveys.

In light of Shannon's insight that unusual events are more informative than common events, we explore some implications of recording asset returns against a chosen quantity of event intensity, which we measure as cumulative unusualness, as opposed to a chosen quantity of time. Exhibit 1 offers an impressionistic visualization of how we rescale calendar time to event time.

Exhibit 1: Calendar Time versus Event Time



The top panel of Exhibit 1 plots the degree of event intensity over eight equal intervals of time. The bottom panel shows the length of time required to reach equal levels of event intensity. In this illustration, we suppose that event intensity requires a threshold of 4. To begin, it takes three periods to reach our threshold for event intensity. Then event intensity occurs after just a single period. And then four periods must pass to again reach the threshold of event intensity. Our proposal is to record returns across equal units of event intensity instead of equal units of time and to carry out investment research in units of event intensity. We next describe how to measure event intensity with empirical precision.

Event intensity

Seemingly, the most natural approach for measuring event intensity would be to monitor the news, record the number and significance of events given some quantification scheme, and carve up history into event units. However, not only would this approach be unduly laborious; it would depend on subjective interpretation with significant potential for bias. Moreover, this approach might overlook non-newsworthy events that nonetheless would be relevant within a particular context. Therefore, we resort to a statistical measure of event intensity based on the Mahalanobis distance.

<u>The Mahalanobis Distance</u>

The Mahalanobis distance was introduced originally in 1927 and modified in 1936 by an Indian statistician to analyze resemblances in human skulls among people with mixed British and Indian parentage.[1] Mahalanobis compared a set of measurements for a chosen skull to the average of those measurements across skulls within a given group. He also compared the co-occurrence of those measurements for a chosen skull to their covariation within the group. He summarized these comparisons in a single number which he used to place a given skull in one group versus another group.

The Mahalanobis distance has since been applied across many different fields, including medicine (Su and Li, 2002; Wang, Su, Chen, and Chen, 2011; and Nasief, Rosado-Mendez, Zagzebshi, and Hall, 2019); engineering (Lin, Khalastchi, and Kaminka, 2010); and finance (Chow, Jacquier, Kritzman, and Lowry, 1999; Czasonis, Kritzman, and Turkington, 2020; Czasonis, Kritzman, and Turkington, 2021; Kinlaw, Kritzman, and Turkington, 2020; and Czasonis, Kritzman, Pamir, and Turkington, 2020).

Perhaps the application that is most relevant to our current proposal to record returns in event time is Chow, Jacquier, Kritzman, and Lowry (1999). They compared a set of asset class returns for a given time interval to their averages and covariances over a prior history to measure the statistical unusualness of that set of returns as an indication of financial turbulence. They reasoned that the more unusual were the returns the more likely it was that they were driven by disruptive events instead of noise, and that they were therefore more characteristic of financial turbulence. However, they stopped short of converting calendar time into event time or explicitly relating unusualness to informativeness.

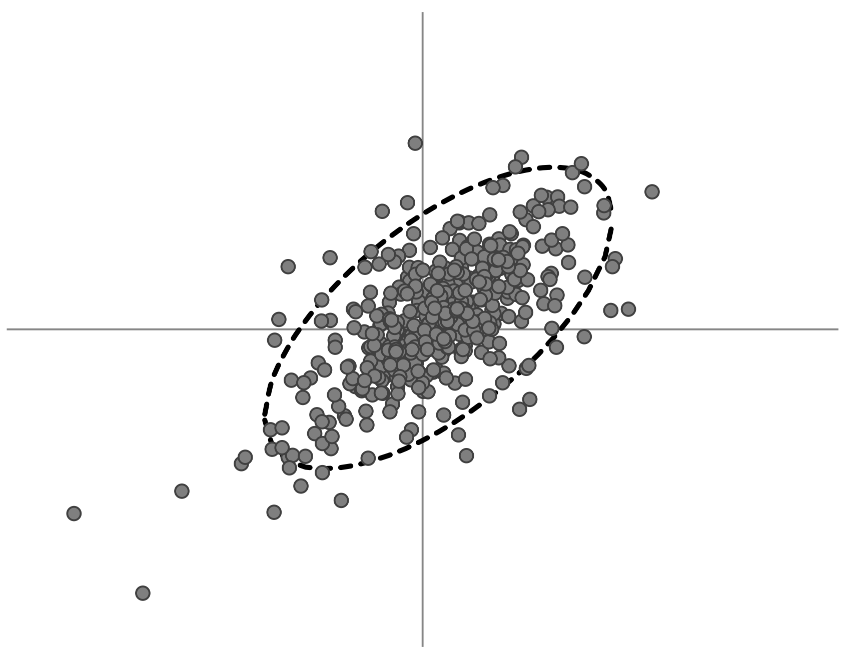
The Mahalanobis distance, as we apply it to measure event intensity, is given by Equation 1.

$$d = (x - \mu)\Sigma^{-1}(x - \mu)' \tag{1}$$

In Equation 1, d equals the Mahalanobis distance, x equals a row vector of values for a set of variables used to characterize a given period, μ equals the average values of the variables measured over a prior window of time, Σ^{-1} equals the inverse of the covariance matrix of the variables over the prior window, and $'$ denotes matrix transpose. The term $(x - \mu)$ captures how similar each variable, by itself, is to the average values. By multiplying $(x - \mu)$ by the inverse of the covariance matrix, the formula captures how similar the co-occurrence of the variables is to their co-occurrence over the prior window. This multiplication also converts the variables into common units. This feature is very handy because some of the variables might be measured as percentage changes whereas others might be measured as levels.

Exhibit 2 helps us to visualize the Mahalanobis distance.

Exhibit 2: Scatter Plot of Two Hypothetical Variables



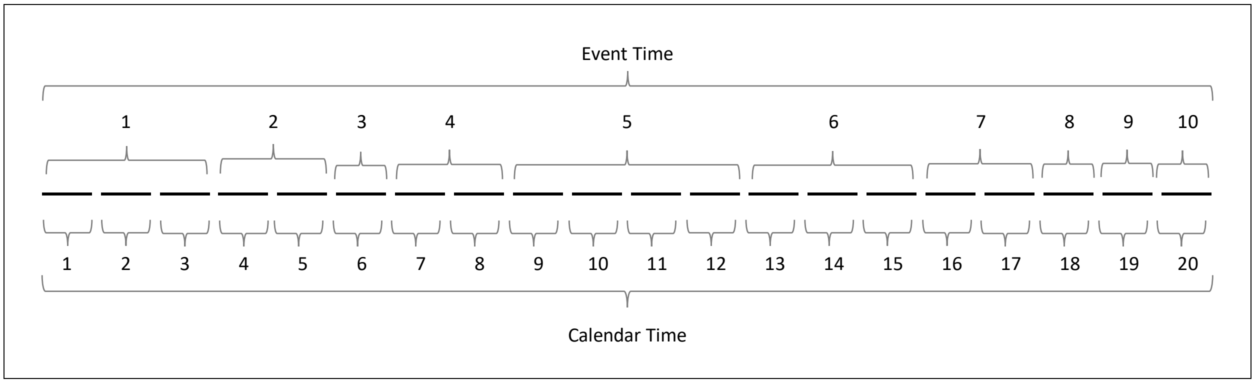
In Exhibit 2 each dot represents the joint values of two variables for a given observation. The center of the ellipse represents the average values of the two variables. The observations within the ellipse represent reasonably common combinations because the observations are not particularly distant from the average values. The observations outside the ellipse are statistically unusual and therefore likely to be driven by events. And, according to information theory, they are also more informative.

Note that observations just outside the narrow part of the ellipse are closer to the ellipse's center than some observations within the ellipse at either end. This illustrates the notion that some observations qualify as unusual not because one or more of the values is unusually distant from the average value (in Euclidean units), but instead because the values have opposite signs relative to the average even though these two variables are positively correlated, as evidenced by the positive slope of the scatter plot. Unusualness as measured by the Mahalanobis distance provides a proper measure of multivariate informativeness.

This description of the Mahalanobis distance assumes that we fix the time interval of the observations to yield various distances. To convert calendar time to event time, we instead fix a threshold for the Mahalanobis distance and compute the Mahalanobis distance for small successive intervals of time until the sum of the Mahalanobis distances computed over these small intervals reaches our threshold for event intensity, at which point we record one unit of event time. We then proceed from the end of the first event period and compute the Mahalanobis distance of the variables over new successive intervals until we again reach the event intensity threshold. We proceed through time until we have accounted for all the small intervals in our sample.

Exhibit 3 helps us to visualize this process.

Exhibit 3: Conversion of Calendar Time to Event Time



The dashed line at the center of Exhibit 2 represents 20 short calendar intervals. For each calendar interval, we calculate the Mahalanobis distance. Then we sum these distances until the sum equals our threshold for event intensity. Initially three calendar intervals are required to reach our event intensity threshold, then two periods, then only one period, and so on. Though there are 20 calendar periods, there are only 10 event periods.

One might consider computing the Mahalanobis distance over an expanding window until it reaches the event intensity threshold, but this approach could yield a misleading result. For example, some of the asset returns in the first period might be significantly above average or they may have co-moved in a way that is contrary to their historical pattern, but not so much to reach the event intensity threshold. Then in the next calendar interval, they might have reversed their pattern such that cumulatively over the two periods it would appear that there were no significant events. By computing the Mahalanobis distance repeatedly over each small calendar interval, we capture offsetting events that are obscured when they are averaged over longer intervals.

Implementation Issues

Thus far we have noted that unusualness corresponds to event intensity, and event intensity corresponds to informativeness. And we showed how to measure event intensity mathematically. But we have not yet addressed an important implementation issue, which is the choice of variables used to indicate event intensity. These variables will differ depending on the context in which we measure event intensity. Suppose, for example, we wish to analyze the U.S. equity market. We might measure event intensity based on the returns of sectors within the U.S. equity market or the returns of asset classes that make up a typical multi-asset portfolio. We may even include economic variables that are believed to affect equity prices. If, instead, our goal is to analyze currencies, it would make more sense to determine event intensity from currency returns and perhaps interest rates across various countries. The choice of variables is not ordained theoretically. Instead, we must rely on empirical analysis, experience, and judgment.

Implementation also requires us to select a threshold for delineating event intensity. This choice as well requires analysis and judgment. The units of the Mahalanobis distance tell us nothing in an absolute sense. It therefore might be helpful to convert the Mahalanobis distance into a measure of likelihood, as shown by Equation 2.

$$likelihood \propto e^{-d/2} \tag{2}$$

In Equation 2, d equals the Mahalanobis distance, e is the base of the exponential function, and $\propto$ denotes a proportionality relationship.

This conversion allows us to select a threshold based directly on the likelihood of occurrence, in line with the precepts of information theory. But it does not obviate the need for analysis and judgment.

We next discuss the implications of event time analysis on statistical inference.

Implications for Statistical Inference

Statistical inference is more reliable to the extent the return samples upon which it is based are well behaved. We first consider this issue from a univariate perspective. We follow this analysis with a novel approach for measuring the bivariate stability of return samples.

Univariate Perspective

The inferences we draw from financial analysis almost always assume, at least as a first approximation, that asset returns expressed in continuous units are normally distributed. This assumption is baked into the Black-Scholes-Merton option pricing formula, determines the confidence we attach to investment strategies, and influences how investors contrive stress scenarios, to name but a few dependencies. It therefore seems appropriate to consider whether the assumption of normality is a better description of returns measured in calendar time or event time. If event returns are distributed more normally than calendar returns, then in some applications it may make more sense to base our inferences on event time.

We measure daily event intensity based on the returns of 11 U.S. equity sectors[2] and their full sample means and covariances over the period March 1, 1994 through November 30,

2021. We define overlapping event periods using an event intensity threshold of 20. Calendar time, therefore, converges to an event unit when the cumulative Mahalanobis distance of daily returns reaches 20. We identify overlapping event periods by rolling the event intensity threshold forward each day. This is analogous to rolling forward a fixed calendar window. However, in the case of event periods, the number of days in the rolling window varies based on cumulative event intensity. We focus on overlapping rather than distinct event periods so that our analysis is not sensitive to the start date. Empirically, the average number of days for an event unit is 33 days, though individual event units will almost always correspond to fewer or greater daily units just as daily calendar returns almost always differ from their average.

We measure asset class returns over these rolling event periods and rolling 33-day calendar windows to match the average number of days per event period. Specifically, we consider the following asset classes:[3]

- U.S. Equities – Ken French U.S. Market
- U.S. Treasury Bonds – Bloomberg Barclays U.S. Treasury Aggregate Index
- U.S. Corporate Bonds – Bloomberg Barclays U.S. Corporates Investment Grade Index
- Commodities – GSCI Commodities Total Return Index
- Multi-Asset Portfolio – 60% U.S. Equities, 30% U.S. Aggregate Bonds (Bloomberg Barclays U.S. Aggregate Bond Index), 10% Commodities

The first two panels of Exhibit 4 show the skewness and kurtosis of asset class returns measured in calendar time and event time. The rightmost panel summarizes the divergence of each empirical distribution from a normal distribution with the same mean and standard deviation. We measure normal divergence using Kullback-Leibler divergence,[4] also called relative entropy, which measures how one probability distribution, Q, differs from a reference probability distribution, P. Typically, Q corresponds to a theoretical distribution and P to a measured distribution. Relative entropy is always non-negative, with zero indicating that the two distributions contain the same quantities of information. Relative entropy greater than zero indicates a loss of information from using Q to model the true distribution, P.

For discrete probability distributions, the relative entropy from Q to P over a set X is defined as:

$$D_{KL}(P \parallel Q) = \sum_{x \in X} P(x) log\left(\frac{P(x)}{Q(x)}\right) \tag{3}$$

For our purposes, we define P as the empirical distribution of calendar or event returns for a given asset class and Q as a normal distribution with the same mean and standard deviation.[5] We interpret Kullback-Leibler divergence as an indication of whether a normal distribution reasonably approximates the empirical return distribution. The greater the divergence, the less normal the empirical distribution.

Exhibit 4: Normality of Asset Class Returns in Calendar Time and Event Time

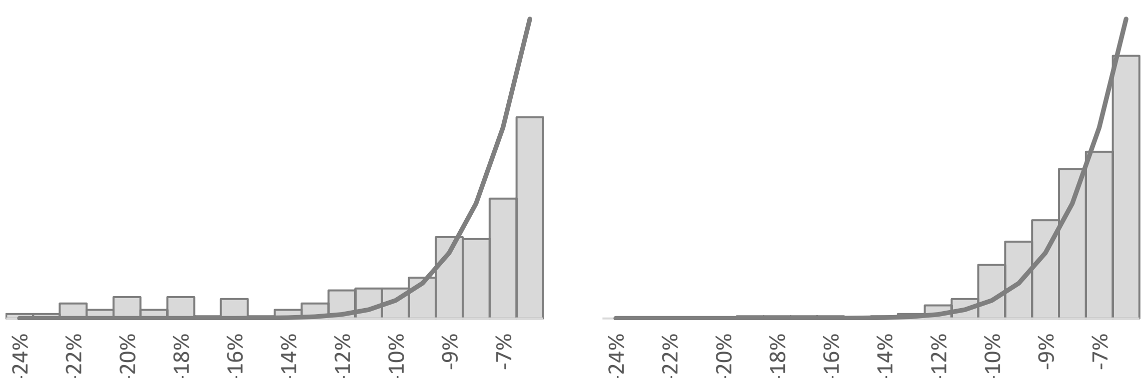
	Skewness		Kurtosis		Normal Divergence	
	Calendar	Event	Calendar	Event	Calendar	Event
U.S. Equities	-0.9	-0.4	6.8	3.4	0.09	0.02
U.S. Treasury Bonds	0.2	-0.1	4.5	3.9	0.02	0.02
U.S. Corporate Bonds	-0.9	-0.1	9.6	3.8	0.09	0.02
Commodities	-0.8	-0.2	5.2	3.1	0.06	0.01
Multi-Asset Portfolio	-1.3	-0.4	8.6	3.5	0.11	0.02

Exhibit 4 starkly reveals that event returns, in which event intensity is estimated from sector returns, are distributed more normally than calendar returns, on balance. Exhibit 5 offers visual support of this observation using the multi-asset portfolio as an example. We plot the left tail of calendar returns (left panel) and event returns (right panel) along with the left tail of the normal distribution.[6] We focus on the left tail to help visualize the differences in negative skewness and fat tails (kurtosis) for calendar and event returns.

Exhibit 5: 5th Percentile Left Tail of Calendar Return and Event Return Distributions: Multi-Asset Portfolio

Calendar Returns

Event Returns



Bivariate Perspective

So far, we have provided persuasive evidence that the returns of major asset classes are much better approximated by a normal distribution when they are measured in event units as opposed to calendar units. Next, we show that the co-movement of asset class returns is more stable when their returns are measured in event units rather than calendar units.

Czasonis, Kritzman, Turkington (2022) introduced a measure of co-occurrence that captures the co-movement of the cumulative returns of two assets over a single period:

$$c_t(x,y) = \frac{\left(\frac{r_{x,t} - \mu_x}{\sigma_x}\right)\left(\frac{r_{y,t} - \mu_y}{\sigma_y}\right)}{\frac{1}{2}\left(\left(\frac{r_{x,t} - \mu_x}{\sigma_x}\right)^2 + \left(\frac{r_{y,t} - \mu_y}{\sigma_y}\right)^2\right)} \tag{4}$$

Here, r_x and r_y equal the cumulative return of two assets, x and y, over a given period; μ_x and μ_y equal their long-run arithmetic average return with the same periodicity; and σ_x and σ_y equal the standard deviation of their returns with the same periodicity.

Co-occurrence relates to the Pearson correlation of a time series of asset returns in a precise mathematical way:

$$\rho(x,y) = \frac{1}{N-1}\sum_{t=1}^{N}(info_t \times c_t) \tag{5}$$

Here, N equals the number of return periods in the sample; $info_t$ is the informativeness of the asset returns ending at time t; and c is their co-occurrence ending at time t. In this context, informativeness is defined as the assets' average squared z-score for the chosen period.

If we re-write $info_t$ as the assets' average squared z-score and substitute Equation 4 for c_t: (6)

$$\rho(x,y) = \frac{1}{N-1}\sum_{t=1}^{N}\left(\frac{1}{2}\left(\left(\frac{r_{x,t}-\mu_x}{\sigma_x}\right)^2+\left(\frac{r_{y,t}-\mu_y}{\sigma_y}\right)^2\right)\times\frac{\left(\frac{r_{x,t}-\mu_x}{\sigma_x}\right)\left(\frac{r_{y,t}-\mu_y}{\sigma_y}\right)}{\frac{1}{2}\left(\left(\frac{r_{x,t}-\mu_x}{\sigma_x}\right)^2+\left(\frac{r_{y,t}-\mu_y}{\sigma_y}\right)^2\right)}\right)$$

This equation simplifies to the assets' full-sample covariance divided by the product of their standard deviations, the formula commonly used to measure time series correlation:

$$\rho(x,y) = \frac{1}{N-1}\sum_{t=1}^{N}\left(\left(\frac{r_{x,t}-\mu_x}{\sigma_x}\right)\left(\frac{r_{y,t}-\mu_y}{\sigma_y}\right)\right) = \frac{Cov(x,y)}{\sigma_x\sigma_y} \tag{7}$$

This equivalence reveals that we can interpret the Pearson correlation as a weighted average of individual co-occurrences. Just as any single return is likely to deviate from its average, so too will individual co-occurrences deviate from their average, or Pearson correlation. It is useful to measure the typical dispersion of co-occurrences around their mean to understand how they may vary from one period to another. To measure the dispersion of co-occurrences for a pair of assets, we estimate the standard deviation of their informativeness-weighted co-occurrences, which are the elements whose average yields the correlation:

$$Dispersion(c) = \sqrt{\frac{1}{N-1}\sum_{t=1}^{N}\left((info_t \times c_t)-\frac{1}{N-1}\sum_{t=1}^{N}(info_t \times c_t)\right)^2} \tag{8}$$

For our purposes, we are interested in whether co-occurrences measured over event periods are more stable, or less disperse, than those measured over calendar periods. Based on rolling event and calendar returns from the previous section, Exhibit 6 reports the dispersion of co-occurrences for each pair of assets. Though individual co-occurrences as defined in Equation 4 are bound between –1 and 1, it is possible for their dispersion, as defined in Equation 8, to extend beyond this range. Mathematically, this results from multiplying the individual co-occurrences by their informativeness before estimating their standard deviation.

Exhibit 6: Dispersion of Co-occurrences for Asset Class Returns in Calendar Time and Event Time

Calendar Returns

		A	B	C	D	E
A	U.S. Equities					
B	U.S. Treasury Bonds	1.3				
C	U.S. Corporate Bonds	2.2	1.6			
D	Commodities	1.9	1.4	2.0		
E	Multi-Asset Portfolio	2.5	1.4	2.5	2.1	

Event Returns

		A	B	C	D	E
A	U.S. Equities					
B	U.S. Treasury Bonds	1.1				
C	U.S. Corporate Bonds	1.0	1.6			
D	Commodities	1.1	1.0	0.9		
E	Multi-Asset Portfolio	1.5	1.2	1.1	1.1	

Exhibit 6 reveals that the dispersion of co-occurrence is uniformly lower across asset class pairs when their returns are measured in event time as opposed to calendar time, which means that correlations measured in event time are more reliable than correlations measured in calendar time.

Now that we have shown that return samples are more normally distributed and their co-occurrences are more stable when measured in event time instead of calendar time, we next discuss three practical implications of our analysis: stress testing, performance evaluation, and portfolio construction.

Practical Implications

Stress Testing

To evaluate the implications of our analysis on stress testing, we reconsider the unusualness of several stock market crashes from the perspective of both calendar time and event time. As before, we use the daily returns of 11 U.S. sectors to determine event intensity. However, for this application, we extend our data to begin in July 1926. This provides a large sample for estimating historical calendar and event returns to contextualize stock market crashes. Moreover, at each point in time, we measure daily event intensity based on means and covariances estimated over a 10-year (2,520-day) lookback window. We use a trailing window, rather than the full sample, to avoid event concentration, which might occur if we based event intensity on full sample means and covariances measured over such a long sample.

Exhibit 7 measures the unusualness of five stock market crashes based on both calendar time and event time: the 1987 crash, the LTCM crash, the Dotcom Bubble, the Global Financial

Crisis, and the COVID crash. For each drawdown, we quantify its unusualness as z-scores based on a distribution composed of (1) historical returns measured over the same calendar horizon and (2) historical returns measured over event periods with the same cumulative event intensity.

Exhibit 7: Market Crash Z-Scores Measured in Calendar Time and Event Time

	1987	LTCM	Dotcom	GFC	Covid
Start date (Peak)	8/25/1987	7/17/1998	3/24/2000	10/9/2007	2/19/2020
End date (Trough)	12/4/1987	10/8/1998	10/9/2002	3/9/2009	3/23/2020
Drawdown	-33%	-22%	-50%	-55%	-34%
Calendar horizon (Days)	72	59	638	356	24
Drawdown Z-score	-4.27	-3.28	-2.89	-3.38	-7.35
Event intensity	261	202	1486	680	225
Drawdown Z-score	-2.02	-1.75	-2.45	-2.59	-2.32

Exhibit 7 clearly shows that the calendar time distribution drastically overstates the rarity of these crashes. For example, the recent COVID crash based on calendar time is more than seven standard deviations away from the average return of similar length periods, which means it is deemed to have had only a 1 in 10 trillion chance of occurrence. By contrast, the same COVID crash event appears to have been much more plausible when compared to prior periods of similar event intensity: it is 2.3 standard deviations away from average, which implies that it is a little more likely than a 1 in 100 chance.

These stark differences in perceived unusualness based on calendar time and event time have significant implications for stress testing a strategy or an institution's resilience to negative shocks. Specifically, these results suggest that severe shocks are far more plausible than calendar time measures of unusualness would suggest, thereby suggesting that survival may require a more conservative approach to risk taking than commonly assumed.

Performance Evaluation

Investors typically evaluate the investment performance of a manager or a strategy based on a fixed amount of elapsed time, such as a quarter or a year. Yet, as we have shown, common units of elapsed time will likely have much different degrees of event intensity. A quarter with one unit of event intensity reveals much less about the quality of a manager or a strategy than a quarter with 10 units of event intensity. It therefore stands to reason that decisions to fund or defund managers or strategies should be based on returns measured in event time rather than calendar time.

Portfolio Construction

To the extent that estimates of volatility and correlation are more stable when measured in units of event time, it may be advantageous to use these estimates as the inputs to mean-variance analysis or other portfolio construction techniques. The resulting portfolios will be designed to withstand event risk regardless of the length of elapsed time it takes for these events to materialize.

Conclusion

We argued that it is more informative to measure investment returns in units of common event intensity instead of units of common elapsed time. We grounded our argument in information theory which implies that the amount of information revealed by a sequence of returns increases with the degree of event intensity associated with those returns. We then showed how we can measure event intensity with a statistic called the Mahalanobis distance, which gives a multivariate indication of the unusualness of a set of variables.

We presented a comparative analysis of the statistical properties of asset class returns based on calendar time and event time. We showed that when returns are measured in event time as opposed to calendar time, they are better described by a normal distribution. We also showed that the co-occurrence of returns is more stable in event time than calendar time, meaning that event time correlations are more reliable than calendar time correlations. This latter result is particularly relevant to portfolio construction.

Finally, we discussed three practical implications of our analysis. We re-estimated the unusualness of several stock market crashes, revealing that their occurrences were much more plausible from the perspective of event time than calendar time. This result suggests that negative shocks are substantially more likely than one might assume from a calendar time perspective; hence, survival may require a more conservative approach to risk taking than assumed by common approaches to stress testing. We also argued that, to receive the same level of information across evaluation units, investors should evaluate investment managers and strategies based on a common degree of event intensity rather than a common amount of elapsed time. And third, we argued that constructing portfolios from inputs measured in units of event time may render those portfolios more resilient to adverse scenarios that occur over different spans of time.

Notes

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[1] See Mahalanobis (1927) and Mahalanobis (1936).

[2] Specifically, we use the Ken French 12 industry portfolios and exclude the industry labeled as “Other.” We obtain this data from the Ken French online data library.

[3] We obtain the U.S. market from the Ken French online data library and all other series from Thomson Reuters Datastream.

[4] See Kullback and Leibler (1951).

[5] Specifically, we partition each empirical distribution into 50 equal-sized bins, X, that cover the full range of observed values and measure the frequency of observations that fall within each, $P(x)$. Then, we estimate the normal frequency of occurrence for each bin, $Q(x)$, as the difference in the cumulative distribution function at the bin’s maximum and minimum values for a normal distribution with the same mean and standard. If the empirical frequency of a bin equals zero, $P(x) = 0$, we set its corresponding term in Equation 3 to zero.

[6] To facilitate visual comparison, we shift and re-scale the distribution of event returns to match the mean and standard deviation of calendar returns. This preserves the shape of the event distribution while allowing for proper comparison with the calendar distribution. We define the 5% left tail according to a normal distribution with the same mean and standard deviation.